Discriminatory Power of the Altman Z-Score Model

Kateřina Mičudová
University of West Bohemia

Abstract

This article aims to assess the discriminatory power of one of the most famous and most discussed corporate predictive models, the Altman Z-Score. This model ranks among the bankruptcy models, whose main purpose is to detect the impending bankruptcy in good time. The research focuses on three main areas of assessing the discriminatory power of the model. The first part deals with the overall discriminatory power of the model; the second part is aimed at quantifying the impact of individual variables on misclassification of enterprises in bankruptcy. The last part quantifies the discriminatory power of individual variables of the model. The results are compared with the findings of the author of the model. The empirical research is based on the accounting data of Czech companies from the manufacturing industry. Both thriving and bankrupt companies are included in the research.

Keywords: Altman, bankruptcy, financial health, predictive model, Z-Score

Introduction

The ability to successfully derive future values of key variables has always belonged with the objects of human interest and has not even avoided the business sector. For several decades, many economists have been trying to find a way how to assess the health of a business as accurately as possible, or predict bankruptcy.

There are hundreds of more or less known methods and models assessing the financial health of companies, possibly predicting the impending bankruptcy. Prof. Altman is undoubtedly one of the most important and best-known analysts in the field of bankrupt companies. He is the author of many prediction models based on various statistical methods. However, his first published index based on multiple discriminant analysis of 1968 (Altman 1968) still remains his most famous model. This model is designed for companies listed on the stock market and therefore Czech companies should use the revised version of the model from 1983, which is also adapted to firms not traded publicly (Altman

2002). The article aims to analyze the discriminatory power of the revised model whose formulation is as follows:

$$Z = 0.717X_1 + 0.847X_2 + 3.107X_3 + 0.420X_4 + 0.998X_5,$$

where X_1 is the working capital/total assets ratio, X_2 is retained earnings/total assets ratio, X_3 is the earnings before interest and taxes (EBIT)/total assets ratio, X_4 is the book value equity/book value of total liabilities ratio, and X_5 is the sales/total assets ratio.

If the resulting value of the Z-Score is higher than 2.9, the firm is financially sound. If the resulting value of the Z-Score is lower than 1.23, the firm is considered failing. Values ranging from 1.23 to 2.9 indicate the grey zone, for which there is no clear prediction.

When comparing the gravity of errors in this revised model and that of 1968, Altman concluded that one year prior to bankruptcy the Type I error (misclassification of a company in bankruptcy) in the revised model is by 3% (percentage points) higher than in the original model (the original model – 6%, the revised model – 9%), and the Type II error (misclassification of going concerns) is identical in both models (3%) (Altman 2002).

The reliability of the Altman models has been verified by the author himself and other analysts many times. Russ et al. (2009) concluded that the accuracy of the Altman model is sufficient. The model was tested on a sample of several thousands of firms. The resulting Type I error was 20.6% and the Type II error was 28.4%. Lacher et al. (1995) is among the following authors who found the accuracy of the modal to be sufficient. The Type I error was 17% and the Type II error was 4.3% in their set of firms. In contrast to it, Boritz et al. (2007), who assessed the reliability of the model in predicting bankruptcy of Canadian companies, found the predictive power of the model insufficient. The model revealed only 41.7% of bankruptcies. The predictive power of the model for Czech companies was tested by e.g. Vochozka (2011), Maňasová (2008) or Kopta (2006). The Czech authors mentioned above came to the conclusion that the Altman Z-Score could unequivocally detect the bankruptcy one year prior to the bankruptcy itself in circa 60% of companies. Circa 10% of companies were erroneously assessed as prosperous. The authors addressed the discriminatory power of the model as a whole, but not the discriminatory power of the individual variables of the model. The determination of key variables that influence the resulting value of the Z-Score is a necessary step to the correct application of the model and, in particular, to the detection of any erroneous predictions.

Material and Methods

Collection and Characteristics of Input Data

Given the nature of the research, the input data are composed of the financial indicators of selected manufacturing firms. The manufacturing industry has been chosen for its dominant position within the Czech economy. Due to the

comparability, firms from other industries have not been included into the analysis. The sample analyzed consists of both prosperous and failing companies. The method of selecting companies corresponds to the selection of enterprises in other professional studies or works.

The group of thriving companies is made up of 47 firms that Čekia, a.s. as well as Coface Czech found successful. An indisputable advantage of these charts is their attempt to assess the overall situation of firms; it is a comprehensive assessment of firms' performance. The analyzed flourishing companies took the highest places in the aforementioned charts in 2009 and 2010. For reasons of comparability, a longer period of time was not taken into account. The financial ratios of firms were monitored in the period from 2008 to 2010.

38 companies were ranked among those in bankruptcy and their financial ratios for the period of 1–3 years prior to the declaration of bankruptcy were monitored. The only condition for including a company in this group was the court decision to declare bankruptcy issued from 2007 to 2011. In order to compare the input data, we did not take a longer period of time into account. The sample of bankruptcy companies was chosen by non-random selection (due to data availability). The bankruptcy of manufacturing companies that published the financial statements for observation periods were included in the sample. The data availability, especially for companies in bankruptcy, is very low. The company Creditreform publishes information on compliance of obligation to publish the financial statements by Czech companies. At the end of 2010, only 21% of limited liability companies and 35% of joint stock companies saved the financial statements for the year 2009 in Collection of Documents. Czech companies include to the worst in Europe (Creditreform 2011). The non-random samples of firms are also used in the construction of the predictive models themselves, for example Altman (1968), Taffler (1983), Ohlson (1980), Altman et al. (1977). Some authors found that if a failure prediction model is estimated on samples that are non-random it may give inefficient predictions (Balcaen and Ooghe 2006; Vochozka 2011). In contrast, Zmijewski (1984) found that non-random samples do not significantly affect the overall accuracy rates.

Albertina, the database of firms and institutions, and the collection of documents were the main sources of firms' financial data.

Methodology for Determining the Discriminatory Power of Variables

The impact of variables on the misclassification of companies in bankruptcy

As the high error rate was not recorded with going concerns, the influence of individual variables on misclassification was analyzed only with bankrupt companies. Companies whose resulting value of the Z-Score is lower than 1.23 ($Z < 1.23$), i.e. they are viewed as bankrupt by the model, are considered to be correctly classified as companies in bankruptcy. Companies that are assessed as prosperous by the model and whose resulting Z-Score is higher than 2.90 ($Z > 2.90$) are considered to be misclassified. Also companies which are ranked in the so-called grey area, i.e. their Z-Score is $1.23 \leq Z \leq 2.90$, are considered

to be misclassified in the period of one year prior to bankruptcy. This condition is based on the assumption that the company predictive model should be able to unambiguously detect failure at least in the period immediately before the bankruptcy itself.

The effect of the i-variable on the misclassification of enterprises in bankruptcy p_i was quantified with the use of the following equation:

$$p_i = \frac{X_{i1} \cdot b_i - X_{i2} \cdot b_i}{Z_1 - Z_2} \cdot 100 = \frac{b_i \cdot (X_{i1} - X_{i2})}{\sum_{i=1}^{5} b_i \cdot (X_{i1} - X_{i2})} \cdot 100,$$

where X_{i1} is the average value of the i-variable of correctly classified companies in bankruptcy, X_{i2} is the average value of the i-variable of erroneously classified companies in bankruptcy, b_i denotes the coefficient of the model i-variable, Z_1 is the average Z-Score of correctly classified companies in bankruptcy, and Z_2 denotes the average Z-Score of misclassified companies in bankruptcy.

Discriminatory power of the model variables

The discriminatory power of the individual variables is determined by two approaches. The first approach, used by Prof. Altman, evaluates the discriminatory power of the variable $i - th$ by its standard deviation σ_i weighted by the coefficient b_i. This approach has been included in the research so that we can compare our results with those of Prof. Altman. However, the assessment of the impact of individual variables with the use of standard deviation may fail in certain situations. This method of assessment assumes that a possible high variability is caused by different values of variables of companies in bankruptcy in comparison with thriving businesses. But that is not the rule. A high standard deviation of a variable caused by a high variability in both groups of companies is not a sign of high discriminatory power, i.e. the ability to distinguish thriving companies from those in bankruptcy.

Therefore, it is preferable to choose a similar way as in assessing the influence of individual variables on the misclassification of companies. This method was also used by Taffler (1983), and Joy and Tollefson (1975). The relative discriminatory power of variables, r_i, is calculated according to the following equation:

$$r_i = \frac{X_{i1} \cdot b_i - X_{i2} \cdot b_i}{Z_1 - Z_2} \cdot 100 = \frac{b_i \cdot (X_{i1} - X_{i2})}{\sum_{i=1}^{5} b_i \cdot (X_{i1} - X_{i2})} \cdot 100,$$

where X_{i1} is the average value of the i-variable of thriving businesses, X_{i2} is the average value of the i-variable of companies in bankruptcy, b_i denotes the coefficient of the model i-variable, Z_1 is the average Z-Score of prosperous businesses, and Z_2 denotes the average Z-Score of companies in bankruptcy.

Results

Classification of Companies

The following table No. 1 shows the classification of successful companies by the Z model in the individual observed years.

Tab. 1: Classification of prosperous firms by the Z model

	Average value of Z-Score (Z)	**Number of firms**		
		$Z < 1.23$	$1.23 \leq Z \leq 2.90$	$Z > 2.90$
2010	3.347	1	15	31
2009	3.521	1	16	30
2008	3.660	2	11	34

Source: author's own elaboration, 2012

In the individual monitored years, the model accuracy in classification of the successful firms, i.e. the ability of the model to assess the thriving firms by the Z-Score value higher than 2.90 ($Z > 2.90$), was ranging from 64% in 2009 to 72% in 2008. Only one company was viewed as bankrupt ($Z < 1.23$) throughout the whole monitored period. In view of the conclusions of other authors (see the introduction of this article) and in view of the fact that an objective business performance criterion cannot be set, we can state that the accuracy of the Z model in classification of thriving companies is sufficient. The majority of firms were classified as thriving or included in the grey zone. Only a negligible percentage of firms were assessed as bankrupt in the individual years. The future development of these firms should be observed.

We expect and require the bankruptcy prediction model to be highly reliable especially when predicting bankruptcy. It is evident from Table No. 2 that the model is less accurate in the classification of firms in bankruptcy than the thriving ones. 1 year prior to the bankruptcy itself, only 55% of firms were classified as those that were definitely at risk of going bankrupt. Two years prior to bankruptcy, only 37% of companies were classified as bankrupt and three years before bankruptcy only 26% of companies analysed were considered bankrupt. Only 8 firms (21%) were viewed as bankrupt in all those observed years. These results cannot certainly be regarded as sufficient. The main purpose of the bankruptcy model is to detect the impending bankruptcy, which the model failed to do so.

The Impact of the Individual Variables on the Misclassification of Companies in Bankruptcy

Table No. 3 which follows illustrates the impact of the individual variables on the misclassification of companies in bankruptcy according to the methodology described above.

Tab. 2: Classification of companies in bankruptcy by the Z model

Number of years prior to bankruptcy	**Average value of Z-Score** (Z)	**Number of companies**		
		$Z < 1.23$	$1.23 \leq Z \leq 2.90$	$Z > 2.90$
1	1.078	21	14	3
2	1.356	14	14	10
3	2.086	10	21	7

Source: author's own elaboration, 2012

Tab. 3: The impact of the variables on the misclassification of companies in bankruptcy (1 year prior to bankruptcy)

	X_1 **(WC/A)**	X_2 **(RE/A)**	X_3 **(EBIT/A)**	X_4 **(BVE/BVTL)**	X_5 **(S/A)**	**Z-score** Z
Misclassification (average values)	-0.177	-0.163	-0.023	0.409	3.010	2.839
Correct classification (average values)	-0.470	-0262	-0.367	0.013	1.348	-0.348
Z **Model – coefficient**	$\times$**0.717**	$\times$**0.847**	$\times$**3.107**	$\times$**0.420**	$\times$**0.998**	$\sum X_i \times b_i$
Misclassification ($X_i \times b_i$ **coefficient**)	-0.127	-0.138	-0.072	0.172	3.004	2.839
Correct classification ($X_i \times b_i$ **coefficient**)	-0.337	-0.222	-1.139	0.005	1.345	-0.348
Effect of variable p_i **(%)**	6.589	2.636	33.480	5.240	52.055	

Source: author's own elaboration, 2012

It is obvious from the data in Table No. 3 that all variables reach higher average values with companies that were misclassified in comparison with the correctly classified ones. 1 year prior to bankruptcy there is the relatively highest difference in average values of the ratio X_4. However, what is really significant for the resulting value of the Z-Score (Z) is the variable value weighted by the coefficient b_i. The variables X_1–X_4 reach low or negative values, so they do not considerably increase the resulting value of the Z-Score (Z). It is evident though that the variable X_3 substantially decreases the resulting value of the Z-score of the correctly classified firms than the misclassified ones. The average values of the variable X_5 of the misclassified companies are above 3 (3 years prior to bankruptcy the values are higher than 4) and thus this variable substantially increases the Z-Score value. It follows that the variable X_5 significantly influences the differences in Z-Scores of correctly and erroneously classified companies in bankruptcy and so it has the greatest influence on the misclassification. We would arrive at the same conclusions if we analyzed the influence of variables on the misclassification of firms in the period of 2 and 3 years prior to bankruptcy. Quantification of the variables influence would be similar to the above-analyzed period of 1 year prior to bankruptcy.

Discriminatory Power of Individual Variables

Altman (1968) studied the significance, impact of the variable $i-th$ using its standard deviation σ_i weighted by the corresponding discriminant coefficient b_i. He calculated the power of individual ratios and came to the conclusion that the ratio X_3 (return on total assets) has the most significant influence on the resulting value. As for our data set of Czech firms, the order of individual variables influence on the Z-score value is the same as in the set of Prof. Altman (only the order of variables X_3 and X_5 was switched). Profitability and total asset turnover most significantly influence the change in the resulting value of the Z-Score. Table No. 4 shows summary results.

Tab. 4: The influence of variables on the change in the Z-Score (Z) (companies in bankruptcy – 1 year prior to bankruptcy and prosperous companies – year 2009)

	standard deviation $-\sigma_i$	$b_i \cdot \sigma_i$	**Order**
X_1 **(WC/A)**	0.500	0.359	5
X_2 **(RE/A)**	0.477	0.404	4
X_3 **(EBIT/A)**	0.299	0.929	2
X_4 **(BVE/BVTL)**	1.993	0.837	3
X_5 **(S/A)**	1.113	1.111	1

Source: author's own elaboration, 2012

However, when assessing the impact of variables on the Z-Score value, it is important for the variable to correctly distinguish bankrupt companies from thriving ones. Therefore, we will focus on the relative discriminatory power

of individual variables. The variable X_4 shows the biggest difference between the average value of thriving and bankrupt companies. However, due to its low coefficient its relative discriminatory power is lower in comparison with the variable X_3. Although X_3 shows the minimum difference in average values, it also shows the highest relative discriminatory power thanks to its high coefficient value, which is close to 50%. Therefore, it greatly exceeds the discriminatory power of the other variables. In comparison with that, the variable X_5 has higher values for companies in bankruptcy than in thriving companies; thereby it has a negative effect on the Z-Score value. It is evident from the analysis performed that if we assessed the discriminatory power of the individual variables by their weighted standard deviation, we would choose X_5 as the variable with the highest discriminatory power. But, in fact, this variable has the lowest ability to classify the thriving and bankrupt companies, and thus the lowest discriminatory power. Table No. 5 illustrates the situation in more detail. In the period of 2 years prior to bankruptcy the relative discriminatory power of individual variables has almost identical values and the order remains unchanged.

Tab. 5: The relative discriminatory power of variables (companies in bankruptcy – 1 year prior to bankruptcy – X_{i2}, and prosperous companies – year 2009 – X_{i1})

	$X_{i1} - X_{i2}$	$b_i(X_{i1} - X_{i2})$	**Relative power of variable (%)**	**Order**
X_1 **(WC/A)**	0.716	0.513	20.999	3
X_2 **(RE/A)**	0.518	0.439	17.970	4
X_3 **(EBIT/A)**	0.389	1.209	49.488	1
X_4 **(BVE/BVTL)**	2.130	0.895	36.635	2
X_5 **(S/A)**	-0.614	-0.613	-25.092	5
$\sum$		2.443	100	

Source: author's own elaboration, 2012

Discussion

The analysis made above has brought several interesting findings. The predictive power of the model in classification of companies in bankruptcy is rather low. Although the Type I error (a bankrupt firm is classified as prosperous) was relatively low (7.4% in the period of 1 year prior to bankruptcy), the bankruptcy was unequivocally detected in the period immediately before the bankruptcy itself only in 55% of cases (in the period of 2 and 3 years prior to bankruptcy the percentage was significantly lower). Many authors hold the view that the reliability of the model is sufficient if the Type I error accounts for 20% (see the introduction of this article); but we may argue that what is really expected and required from the model predicting bankruptcy is its high reliability just when predicting bankruptcy and hence 55% of detected bankruptcies in the period immediately before bankruptcy cannot be considered to be sufficient.

A high value of total asset turnover ratio of companies in bankruptcy is another important finding. The value of this ratio for firms in bankruptcy is even higher than for thriving firms (However, the Z-model assumes that the total asset turnover decreases with the increasing probability of bankruptcy). Some foreign authors in their analysis of variables of foreign firms have come to the same conclusion. Wu, Gaunt and Gray (2010) analysed values of selected variables of 887 American companies, which went bankrupt in the period from 1980 to 2006, and compared them with the values of thriving companies. The asset turnover of companies in bankruptcy was 1.35, while of the thriving companies 1.22. Ooghe and Balcaen (2007) adapted coefficients of the Altman Z-model to the conditions of Belgian firms. A negative value of the coefficient was assigned to the asset turnover variable, which proves a higher value of this ratio of Belgian firms in bankruptcy in comparison with the thriving ones. In the Prof. Altman's set (1968) the bankrupt companies had lower values of asset turnover on the average than the thriving firms but the difference was not statistically significant. A relatively high asset turnover of companies in bankruptcy may be caused by an effort of these companies to avert bankruptcy and obtain the necessary financial means by selling its assets.

Conclusion

Several conclusions have followed from the performed research aimed at the analysis of the discriminatory power of the selected predictive model of Prof. Altman. The model accuracy in classification of thriving companies is sufficient. We expect and require the bankruptcy prediction model to be highly reliable especially when predicting bankruptcy. And just in the case of classification of bankruptcy companies the model fails.

The asset turnover variable has the most significant effect on the misclassification of companies in bankruptcy. The asset profitability variable also has significant influence. As already mentioned in the introduction, the determination of key variables that influence and reduce the accuracy of the model is a necessary step to the correct application of the model and, in particular, to the detection of any erroneous predictions.

The asset turnover and profitability variables have the most significant influence on the Z-Score change. The asset profitability variable has the highest relative discriminatory power (the ability to correctly differentiate companies in bankruptcy from the prosperous ones). On the contrary, the assets turnover variable has the lowest relative discriminatory power. This variable has an opposite effect on the resulting value of the Z-Score. The value of this ratio for companies in bankruptcy is even higher than for thriving companies. Further research may be focused on assessing whether this finding is typical only for manufacturing companies or also for companies of other industries.

Therefore, it is obvious that high variability of assets profitability along with the relatively high value of coefficient significantly influence the discriminatory power of this variable but it causes the misclassification of firms as well. The whole model is then significantly affected by the single variable. Further

possible direction of the research could be focused to determine whether this phenomenon is also typical for other prediction models.

Considering the limited size of the sample of companies, which arised from the requirement to preserve the data homogeneity, it would be possible to continue the research and apply outlined procedures to other companies file structure or companies of other branches, so that the results could be compared and submitted to further discussion.

Reference

ALTMAN, E. I., 1968. Financial ratios, discriminant analysis and the prediction of corporate bankruptcy. *The Journal of Finance.* **XXIII**(4), 589–609. ISSN 0022-1082.

ALTMAN, E. I., 2002. *Bankruptcy, credit risk, and high yield junk bonds.* Malden: Blackwell Publishers Inc. ISBN 0-631-22563-3.

ALTMAN, E. I., R. G. HALDEMAN and P. NARAYANAN, 1977. Zeta analysis. A new model to identify bankruptcy risk of corporations. *Journal of Banking and Finance.* **1**(1), 29–54. ISSN 0378-4266.

BALCAEN, S. and H. OOGHE, 2006. 35 years of studies on business failure: An overview of the classic statistical methodologies and their related problems. *The British Accounting Review.* **38**(1), 63–93. ISSN 0890-8389.

BORITZ, E. J., D. B. KENNEDY and J. Y. SUN, 2007. Predicting business failures in Canada. *Accounting Perspectives.* **6**(2), 141–165. ISSN 1911-3838.

JOY, M. O. and J. O. TOLLEFSON, 1975. On the financial applications of discriminant analysis. *The Journal of Financial and Quantitative Analysis.* **10**(5), 723–739. ISSN 0022-1090.

KOPTA, D., 2006. Metody predikce finanční tísně u zemědělských podniků. In *MVD 2006: Konkurenceschopnosť v EÚ – výzva pre krajiny, Nitra, 17.-18.5. 2006.* Nitra: SPU v Nitre, 1059–1067. ISBN 80-8069-704-3.

LACHER, R. C., P. K. COATS, S. C. SHARMA and F. L. FANT, 1995. A neural network for classifying the financial health of a firm. *European Journal of Operational Research.* **85**(1), 53–65. ISSN 0377-2217.

MAŇASOVÁ, Z., 2008. *Úpadky podniků v České republice a možnosti jejich včasné predikce.* Praha. Doktorská disertační práce. Vysoká škola ekonomická v Praze.

MENŠÍKOVÁ, S., 2011. Umíte správně ukládat účetní závěrky? In *Creditreform* [online]. 1. 2. 2011 [cit. 28-04-2012]. Available from: http://web.creditreform.cz/cs/resources/pdf/20110224_Jak_ukladat_zaverky.pdf

OHLSON, J. A., 1980. Financial ratios and the probabilistic prediction of bankruptcy. *Journal of Accounting Research.* **18**(1), 109–131. ISSN 0021-8456.

OOGHE, H. and S. BALCAEN, 2007. Are failure prediction models widely usable? An empirical study using a Belgician dataset. *Multinational Finance Journal.* **11**(1&2), 33–76. ISSN 1096-1879.

RUSS, R. W., W. W. ACHILLES and A. C. GREENFIELD, 2009. The Altman z-score revisited. *Journal of International Finance and Economics.* **9**(4), 59–73. ISSN 1555-6336.

TAFFLER, R. J., 1983. The assessment of company solvency and performance using a statistical model. *Accounting and Business Research.* **13**(52), 295–307. ISSN 0001-4788.

VOCHOZKA, M., 2011. *Metody komplexního hodnocení podniku.* 1st edition. Praha: Grada. ISBN 978-80-247-3647-1.

WU, Y., C. GAUNT and S. GRAY, 2010. A comparison of alternative bankruptcy prediction models. *Journal of Contemporary Accounting & Economics.* **6**(1), 34–45. ISSN 1815-5669.

ZMIJEWSKI, M. E., 1984. Methodological issues related to the estimation of financial distressprediction models. *Journal of Accounting Research.* **22**(Suppl.), 59–86. ISSN 0021-8456.

Diskriminační síla Altmanova bankrotního predikčního modelu

Cílem článku je posouzení diskriminační síly jednoho z nejznámějších a nejdiskutovanějších podnikových predikčních modelů, Altmanova Z-score. Tento model se řadí mezi bankrotní modely, jejichž hlavním úkolem je včasné odhalení nastupujícího úpadku. Výzkum se zaměřuje na tři základní oblasti posouzení diskriminační síly modelu. První část se věnuje celkové diskriminační síle modelu, druhá část je zaměřena na kvantifikaci vlivu jednotlivých proměnných na chybnou klasifikaci podniků v úpadku. Obsahem poslední části je vyčíslení diskriminační síly jednotlivých proměnných modelu. Výsledky budou porovnány se závěry autora modelu. Empirický výzkum vychází z účetních dat českých podniků zpracovatelského průmyslu. Do zkoumání jsou zahrnuty jednak prosperující podniky a jednak podniky v úpadku.

Klíčová slova: Altman, finanční zdraví, predikční model, úpadek, Z-score

Kontaktní adresa:
Ing. Kateřina Mičudová, Ph.D., Katedra ekonomie a kvantitativních metod, Fakulta ekonomická, Západočeská univerzita v Plzni, Husova 11, 306 14 Plzeň, *e-mail: pitrova@kem.zcu.cz*

MIČUDOVÁ, K. Discriminatory power of the Altman Z-Score model. *Littera Scripta.* 2013, **6**(1), 95–106. ISSN 1805-9112.